

## A one line of ELI to count the number of heads in a sequence of coin tossing

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ELI is an *interactive programming language system* with its *core* derived from the classical *array programming language APL* invented by Ken Iverson. Thus, ELI has 80 *monadic* (i.e. having a right argument only) and *dyadic* (i.e. having a left and a right argument) *primitive* (i.e. provided by the system) *functions* together with 6 *operators* (they apply to primitive functions), each one of them is represented *symbolically* by one or two ASCII characters. For example,  $a < b$  is the *less than* function while  $a <. b$  is the *encode* function, and  $a <- b$  is the *assign* function.

A *line* of ELI code consists of one or several *expressions* separated by a ‘;’ and separate expressions are *executed* from *left to right* as in other programming languages. However, each expression is *evaluated* from *right to left* with all functions having *equal precedence* while an argument to a function inside a pair of parenthesis is evaluated first before the execution of the function involved. For example, we have

```
      3*2+4
18
      (3*2)+4
10
```

The line you enter is the line with indentation and the system responds with a line without indentation. The result of one function can *feed* as an *argument* to the next function or *be assigned* to a *variable* for further use.

Now, let us consider a problem: *throw a coin 1000 times and count the number of heads* turned up in this experiment. We use a 0 to represent a *tail* and a 1 to represent a *head* while a *throw* is simulated by a *random number generator* which produces a 0 or 1 with equal chance. ELI has a *system variable* called `[]IO` which is set to 1. The monadic primitive function `!n` generates  $n$  consecutive integers starting from `[]IO` and `[]IO` can be changed to 0 if we like. We see that

```
      !10
1 2 3 4 5 6 7 8 9 10
      []IO<-0
      !10
0 1 2 3 4 5 6 7 8 9
```

ELI has another primitive function `roll ?n` which *randomly* picks a number (with equal probability) from `!n` each time it is executed. We see that

```
      ?.10
1
      ?.10
7
      ?.2
0
```

Now one prominent feature of APL/ELI is that for a class of primitive functions called *scalar functions* (includes all arithmetic functions) which operates on an *array* is the same as it operates on each element of the array. In particular, when an array is a *vector* (i.e. a 1-dimensional array), we have for the *inverse* function `%a` the following

```
      %2 5 8 10 3
0.5 0.2 0.125 0.1 0.3333333333
      2*!10
0 2 4 6 8 10 12 14 16 18
```

It happens that the *roll* function `?` is also a scalar function:

```

      ?.3 8 10
1 4 8
      ?.2 2 2
0 0 1

```

Now for a vector  $v$ , we can sum it up by applying the **reduction operator** `/` to the *addition* function `+` to its right and the reduction operator can also apply to other suitable primitive functions such as *multiplication*:

```

      +/1 4 8
13
      +/0 0 1
1
      */1 4 8
32
      */0 0 1
0

```

We are almost there. The question now is how to generate a thousand 2's to the right of the roll function `?` before we apply the plus reduction to the result of roll.

Each array  $a$  in ELI has a **shape**  $s$  which is a vector consisting of lengths of each dimension of  $a$ . For a matrix  $a$  its *shape* is a vector of two elements indicating the *height* and the *width* of  $a$ . For a vector  $v$  the shape of  $v$  is simply its length. For an array  $a$ , applying the primitive function **shape** `#` to it will yield its shape. In case  $a$  is a 3 by 4 matrix we have:

```

      a
0 1 2 3
4 5 6 7
8 9 10 11
      #a
3 4
      #8 9 10 11 101 5
6

```

There is a *dyadic* primitive function **reshape** `s#a` which results in an array  $b$  of shape  $s$  using elements from  $a$  (Here we notice another unique feature of ELI inherited from APL that many *primitive function symbols* can represent a *dyadic* or *monadic* function depending on whether it has a left argument):

```

      3 4#!10
0 1 2 3
4 5 6 7
8 9 0 1
      5#8 9 10 11 101 5
8 9 10 11 101
      10#2
2 2 2 2 2 2 2 2 2

```

We observe that if there are not enough elements in  $a$  to form the required  $b$ , *reshape* will reuse the elements in  $a$  and if there are more elements in  $a$  than what is required then a tail section of  $a$  will be dropped. In any case the one line ELI code to solve our problem is as follows:

```

+/?..1000#2

```

We would like to save this piece of code into a *user defined function*. A **defined function**  $f$  in ELI has a *name* (in contrast to a *primitive* function being denoted by a *symbol*), a right argument or a left-right argument pair (or no argument at all). For a simple defined function, we can write it in **short-function form** which is a line of code preceded by the function name and enclosed in a pair of curly brackets where  $x$  is assumed to be the right argument,  $y$  is the left argument if present, and  $z$  or value of the last expression is the return result of the function  $f$ . In our case, we have

```
{cnt_hds: []IO<-0; +/?..x#2}
cnt_hds
```

where the right argument  $x$  is the number of independent throws of a coin. The second line above is ELI's response to a successful definition of a short function named `cnt_hds`. Note that it is important to set `[]IO` to 0 first because the system default value for `[]IO` is 1. Now, we can call the function with different arguments:

```
cnt_hds 1000
501
cnt_hds 2000
987
cnt_hds 8000
3981
501%1000
0.501
981%2000
0.4905
3981%8000
0.497625
```

We notice that the first call above yields a slight different value than what we have earlier (501 vs 502). This is due to the fact that two different sequences of throws are two different experiments. We also observe that the number of total heads divided by the number of throws is fairly close to 0.5 indicating that the coin throw is *fair*.

We used this one line ELI code to illustrate ELI's **primitive-based array-oriented programming**, and its **dataflow style** of coding, i.e. the output of one operation is used as an input to the next operation. The resulting *succinctness* naturally leads to *code clarity* and *programming productivity*. To learn more about ELI and to download the free software please visit <http://fastarray.appspot.com> or <http://www.sable.mcgill.ca/~hanfeng.c/eli/>.